

Spacetime

Davoud Kamani

Institute for Studies in Theoretical Physics and Mathematics (IPM)

P.O.Box: 19395-5531, Tehran, Iran

e-mail: kamani@theory.ipm.ac.ir

Abstract

In this paper we study the effects of noncommutativity on a closed superstring propagating in the spacetime that is compactified on tori. The effects of compactification and noncommutativity appear in the momentum, quantization, supercurrent, super-conformal generators and in the boundary state of the closed superstring emitted from a D_p -brane with the $NS\otimes NS$ background B -field.

PACS numbers: 11.25.-w

Keywords: String theory; D-branes; Noncommutativity.

1 Introduction

There have been much activities exploring the relation between string theory and noncommutative geometry [1, 2, 3]. There have been attempts to explain noncommutativity on D-brane through the study of open strings in the presence of background fields [3].

D-brane can be alternatively described by the “boundary state” of the closed string channel description [4, 5, 6, 7, 8, 9, 10]. The boundary state can be interpreted as a source for a closed string emitted by a D-brane. By introducing the background field $B_{\mu\nu}$ in the string σ -model one obtains mixed boundary conditions for strings. Mixed boundary conditions have been used for studying many properties of D-branes in the background fields [5, 8, 9, 10].

We proceed to study closed superstrings in the spacetime which is compactified on tori. In a special coordinate system, which we call it “closed string frame”, compactification reveals the noncommutativity of the spacetime through closed strings. In other words, by using the boundary state for a closed superstring, we observe the noncommutativity effects on a D_p -brane with background field wrapped on tori. We shall see that, $O(10, 10; \mathbf{R})$ duality group relates coordinates of the spacetime and of the closed string frame.

In the first part of this paper, we study the closed superstring, its T-duality, its quantization and the noncommutativity effects on it in the noncommutative compact spacetime. In the second part, the boundary state equations of closed superstring will be studied.

2 T-duality and closed string frame

The solution of the equation of motion extracted from the string action can be written as the following mode expansion

$$X^\mu(\sigma, \tau) = X_L^\mu(\tau + \sigma) + X_R^\mu(\tau - \sigma) , \quad (1)$$

$$\begin{aligned} X_L^\mu(\tau + \sigma) &= \xi_L^\mu + 2\alpha' \Pi_L^\mu(\tau + \sigma) + \frac{i}{2} \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \tilde{a}_n^\mu e^{-2in(\tau + \sigma)} , \\ X_R^\mu(\tau - \sigma) &= \xi_R^\mu + 2\alpha' \Pi_R^\mu(\tau - \sigma) + \frac{i}{2} \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} a_n^\mu e^{-2in(\tau - \sigma)} . \end{aligned} \quad (2)$$

In the non-zero constant background B -field we can write

$$\begin{aligned} \xi_L^\mu &= (1 + 2\pi\alpha' B)^\mu{}_\nu x_L^\nu , \\ \xi_R^\mu &= (1 - 2\pi\alpha' B)^\mu{}_\nu x_R^\nu , \end{aligned} \quad (3)$$

$$\begin{aligned}\Pi_L^\mu &= (1 + 2\pi\alpha' B)^\mu{}_\nu p_L^\nu, \\ \Pi_R^\mu &= (1 - 2\pi\alpha' B)^\mu{}_\nu p_R^\nu,\end{aligned}\tag{4}$$

$$\begin{aligned}\tilde{a}_n^\mu &= (1 + 2\pi\alpha' B)^\mu{}_\nu \tilde{\alpha}_n^\nu, \\ a_n^\mu &= (1 - 2\pi\alpha' B)^\mu{}_\nu \alpha_n^\nu,\end{aligned}\tag{5}$$

where $B^\mu{}_\nu = g^{\mu\lambda} B_{\lambda\nu}$ and the metric $g_{\mu\nu}$ also is constant. Again the equations (1) and (2) satisfy the equation of motion. Assume that all directions of the spacetime are compactified on tori. According to the equations (3)-(5) we can write

$$\begin{aligned}X^\mu &= \bar{X}^\mu + 2\pi\alpha' B^\mu{}_\nu \bar{X}'^\nu, \\ X'^\mu &= \bar{X}'^\mu + 2\pi\alpha' B^\mu{}_\nu \bar{X}^\nu,\end{aligned}\tag{6}$$

where X'^μ , the T-dual coordinate of X^μ , has definition

$$T_\mu : X^\mu(\sigma, \tau) \rightarrow X'^\mu(\sigma, \tau) = X_L^\mu - X_R^\mu,\tag{7}$$

and \bar{X}^μ and its T-dual coordinate \bar{X}'^μ are

$$\begin{aligned}\bar{X}^\mu(\sigma, \tau) &= \bar{X}_L^\mu(\tau + \sigma) + \bar{X}_R^\mu(\tau - \sigma), \\ \bar{X}'^\mu(\sigma, \tau) &= \bar{X}_L^\mu(\tau + \sigma) - \bar{X}_R^\mu(\tau - \sigma).\end{aligned}\tag{8}$$

Therefore, the mode expansions of the left and the right moving parts of \bar{X}^μ and \bar{X}'^μ are

$$\begin{aligned}\bar{X}_L^\mu(\tau + \sigma) &= x_L^\mu + 2\alpha' p_L^\mu(\tau + \sigma) + \frac{i}{2} \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2in(\tau + \sigma)}, \\ \bar{X}_R^\mu(\tau - \sigma) &= x_R^\mu + 2\alpha' p_R^\mu(\tau - \sigma) + \frac{i}{2} \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{n} \alpha_n^\mu e^{-2in(\tau - \sigma)}.\end{aligned}\tag{9}$$

Let us denote the coordinate system $\{\bar{X}^\mu\}$ as “closed string frame”. These coordinates and their T-dual coordinates can be written as

$$\bar{X}^\mu = G^{\mu\nu} X_\nu + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} X'_\nu,\tag{10}$$

$$\bar{X}'^\mu = G^{\mu\nu} X'_\nu + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} X_\nu,\tag{11}$$

where there are $X_\mu = g_{\mu\nu} X^\nu$ and $X'_\mu = g_{\mu\nu} X'^\nu$. Therefore, the coordinates of the closed string frame and their T-dual are linear combinations of $\{X^\mu\}$ and their T-dual coordinates

$\{X'^\mu\}$. The coefficients of combinations are the elements of the open string metric and the noncommutativity parameter

$$\begin{aligned} G^{\mu\nu} &= \left((g + 2\pi\alpha' B)^{-1} g (g - 2\pi\alpha' B)^{-1} \right)^{\mu\nu}, \\ \theta^{\mu\nu} &= -(2\pi\alpha')^2 \left((g + 2\pi\alpha' B)^{-1} B (g - 2\pi\alpha' B)^{-1} \right)^{\mu\nu}. \end{aligned} \quad (12)$$

For the ordinary spacetime i.e., $\theta^{\mu\nu} = 0$, there are $\bar{X}^\mu = X^\mu$ and $\bar{X}'^\mu = X'^\mu$. Also for the strong background B -field, \bar{X}^μ is a linear combination of the T-dual coordinates $\{X'^\mu\}$. In this case \bar{X}'^μ appears as a linear combination of $\{X^\mu\}$. In other words, we have

$$\begin{aligned} \bar{X}^\mu &= \frac{1}{2\pi\alpha'} (B^{-1})^{\mu\nu} X'_\nu, \\ \bar{X}'^\mu &= \frac{1}{2\pi\alpha'} (B^{-1})^{\mu\nu} X_\nu. \end{aligned} \quad (13)$$

In the zero slope limit [1] i.e., when α' and the closed string metric $g_{\mu\nu}$ go to zero like $\alpha' \sim \epsilon^{1/2}$, $g_{\mu\nu} \sim \epsilon$, where $\epsilon \rightarrow 0$, and $B_{\mu\nu}$ is fixed, we also obtain the above equations.

From the equations (10) and (11) we obtain

$$\begin{aligned} x^\mu &= G^{\mu\nu} \xi_\nu + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} \xi'_\nu, \\ x'^\mu &= G^{\mu\nu} \xi'_\nu + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} \xi_\nu, \end{aligned} \quad (14)$$

$$\begin{aligned} \alpha_n^\mu &= (G^{-1} - \frac{1}{2\pi\alpha'} \theta)^{\mu\nu} a_{n\nu}, \\ \tilde{\alpha}_n^\mu &= (G^{-1} + \frac{1}{2\pi\alpha'} \theta)^{\mu\nu} \tilde{a}_{n\nu}, \end{aligned} \quad (15)$$

where there are $x^\mu = x_L^\mu + x_R^\mu$, $x'^\mu = x_L^\mu - x_R^\mu$, $\xi^\mu = \xi_L^\mu + \xi_R^\mu$ and $\xi'^\mu = \xi_L^\mu - \xi_R^\mu$.

Imposing the worldsheet supersymmetry in the equation (10) or (11) gives the following relations for the worldsheet fermions

$$\begin{aligned} \bar{\psi}^\mu &= G^{\mu\nu} \psi_\nu + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} \psi'_\nu, \\ \bar{\psi}'^\mu &= G^{\mu\nu} \psi'_\nu + \frac{1}{2\pi\alpha'} \theta^{\mu\nu} \psi_\nu, \end{aligned} \quad (16)$$

where $\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$ is worldsheet spinor and $\psi'^\mu = \begin{pmatrix} -\psi_-^\mu \\ \psi_+^\mu \end{pmatrix}$ is its T-dual spinor. Similar definitions also hold for the spinors $\bar{\psi}^\mu$ and $\bar{\psi}'^\mu$. For the ordinary spacetime i.e., $\theta^{\mu\nu} = 0$, we have $\bar{\psi}^\mu = \psi^\mu$ and $\bar{\psi}'^\mu = \psi'^\mu$. For the strong background B -field and also in the zero slope limit, the spinors $\{\bar{\psi}^\mu\}$ are equivalent to the spinors $\{\psi'^\mu\}$ and also $\{\bar{\psi}'^\mu\}$ are equivalent to

$\{\psi^\mu\}$. Note that ψ^μ and $\bar{\psi}^\mu$ satisfy the equation of motion i.e., $\partial_+\psi_-^\mu = \partial_-\psi_+^\mu = \partial_+\bar{\psi}_-^\mu = \partial_-\bar{\psi}_+^\mu = 0$. Furthermore, we have $\psi_{\pm\mu} = g_{\mu\nu}\psi_{\pm}^\nu$.

Assume that all coordinates $\{X^\mu\}$ are compacted on tori with radii $\{R_\mu\}$ therefore,

$$X^\mu(\sigma + \pi, \tau) - X^\mu(\sigma, \tau) = 2\pi\Lambda^\mu, \quad (17)$$

$$\Lambda^\mu = \alpha'(\Pi_L^\mu - \Pi_R^\mu) = n^\mu R_\mu, \quad (\text{no sum on } \mu). \quad (18)$$

In this case the momentum of the closed string is quantized i.e.,

$$\Pi^\mu = \Pi_L^\mu + \Pi_R^\mu = \frac{m^\mu}{R_\mu}. \quad (19)$$

The integers n^μ and m^μ are winding number and momentum number of the closed string around the compact direction X^μ . Therefore, we have the identification

$$\xi^\mu \equiv \xi^\mu + 2\pi\Lambda^\mu. \quad (20)$$

Note that the dual coordinate X'^μ also is compact

$$X'^\mu(\sigma + \pi, \tau) - X'^\mu(\sigma, \tau) = 2\pi\alpha'\Pi^\mu = 2\pi m^\mu \frac{\alpha'}{R_\mu}, \quad (21)$$

therefore, its compactification is on a circle with radius α'/R_μ . This compactification gives the identification

$$\xi'^\mu \equiv \xi'^\mu + 2\pi\alpha'\Pi^\mu. \quad (22)$$

From the identifications (20) and (22) we obtain the following identifications

$$\begin{aligned} x^\mu &\equiv x^\mu + 2\pi L^\mu, \\ x'^\mu &\equiv x'^\mu + 2\pi\alpha' p^\mu. \end{aligned} \quad (23)$$

These imply that the coordinates \bar{X}^μ and \bar{X}'^μ also are compact. The equation (10) or (11) gives L^μ and p^μ as

$$\begin{aligned} L^\mu &= G^{\mu\nu}\Lambda_\nu + \frac{1}{2\pi}\theta^{\mu\nu}\Pi_\nu, \\ p^\mu &= G^{\mu\nu}\Pi_\nu + \frac{1}{2\pi\alpha'^2}\theta^{\mu\nu}\Lambda_\nu. \end{aligned} \quad (24)$$

Therefore, L^μ and p^μ are linear combinations of the closed string momentum numbers and winding numbers. The fact that L^μ depends on the momentum numbers and p^μ depends on

the winding numbers, are consequences of the noncommutativity. Under the T-duality there is the exchange $\Pi^\mu \leftrightarrow \frac{1}{\alpha'} \Lambda^\mu$, which leads to the exchange

$$T_\mu : p^\mu \leftrightarrow \frac{1}{\alpha'} L^\mu . \quad (25)$$

The light-cone components of the worldsheet supercurrent have the forms

$$\begin{aligned} J_+ &= G_{\mu\nu} \bar{\psi}_+^\mu \partial_+ \bar{X}^\nu , \\ J_- &= G_{\mu\nu} \bar{\psi}_-^\mu \partial_- \bar{X}^\nu . \end{aligned} \quad (26)$$

Also the superconformal generators for the $\mathbf{R} \otimes \mathbf{R}$ sector of superstring are

$$\begin{aligned} L_m^{(\alpha, d)} &= \frac{1}{2} \sum_{n \in \mathbb{Z}} G_{\mu\nu} : \alpha_{m-n}^\mu \alpha_n^\nu : + \frac{1}{4} \sum_{n \in \mathbb{Z}} (2n - m) G_{\mu\nu} : d_{m-n}^\mu d_n^\nu : + \frac{5}{8} \delta_{m,0} , \\ F_m^{(\alpha, d)} &= \sum_{n \in \mathbb{Z}} G_{\mu\nu} \alpha_{-n}^\mu d_{m+n}^\nu . \end{aligned} \quad (27)$$

The open string metric explicitly appears in these operators. Similar relations also hold for the left parts of the virasoro operators i.e., for $\tilde{L}_m^{(\tilde{\alpha}, \tilde{d})}$ and $\tilde{F}_m^{(\tilde{\alpha}, \tilde{d})}$, and for the $\mathbf{NS} \otimes \mathbf{NS}$ sector.

$O(10, 10; \mathbf{R})$ Duality relation

Now we discuss $O(10, 10; \mathbf{R})$ duality relation between the spacetime and the closed string frame. In this subsection, let the metric $g_{\mu\nu}$ be Euclidean. For d -dimensional toroidal compactification duality group is $O(d, d; \mathbf{R})$ [11, 12]. The elements $h \in O(d, d; \mathbf{R})$ preserve the form of the matrix J i.e.,

$$h^T J h = J \equiv \begin{pmatrix} \mathbf{0} & \mathbf{1}_{d \times d} \\ \mathbf{1}_{d \times d} & \mathbf{0} \end{pmatrix} . \quad (28)$$

Define 20-dimensional vectors \bar{R} and R as $\bar{R}^m = \begin{pmatrix} \bar{X} \\ \bar{X}' \end{pmatrix}$ and $R^m = \begin{pmatrix} X \\ X' \end{pmatrix}$. The equations (10) and (11) imply that these vectors to have the relation $\bar{R}^m = \Gamma^{mn} R_n$, where $R_m = \bar{\mathcal{G}}_{mn} R^n$. The 20×20 matrices Γ^{mn} and $\bar{\mathcal{G}}_{mn}$ are

$$\Gamma^{mn} = \begin{pmatrix} G^{-1} & \frac{1}{2\pi\alpha'} \theta \\ \frac{1}{2\pi\alpha'} \theta & G^{-1} \end{pmatrix} , \quad \bar{\mathcal{G}}_{mn} = \begin{pmatrix} g & 0 \\ 0 & g \end{pmatrix} , \quad (29)$$

where $m, n \in \{1, 2, \dots, 20\}$. The equations (10) and (11) also give \bar{R}_m as $\bar{R}_m = \Gamma_{mn} R^n$, where the matrix Γ_{mn} is

$$\Gamma_{mn} = \mathcal{G}_{mp} \Gamma^{pq} \bar{\mathcal{G}}_{qn} , \quad \mathcal{G}_{mn} = \begin{pmatrix} G & 0 \\ 0 & G \end{pmatrix} . \quad (30)$$

The open string metric G is, $G_{\mu\nu} = (g - (2\pi\alpha')^2 B g^{-1} B)_{\mu\nu}$. Using the identity $G^{-1} - \frac{1}{(2\pi\alpha')^2} \theta G \theta = g^{-1}$, we observe that, the matrix Γ is orthogonal and belongs to the duality group $O(10, 10; \mathbf{R})$. That is $(\Gamma^T)^{mp} \Gamma_{pn} = \delta^m_n$ and $(\Gamma^T)^{mp} J_p^q \Gamma_{qn} = J^m_n$. In fact, these imply that the closed string frame and the spacetime are related to each other by $O(10, 10; \mathbf{R})$ duality rotation.

The square length of the vector \bar{R} and the inner product $G_{\mu\nu} \bar{X}^\mu \bar{X}^\nu$ are preserved. Therefore, for the left and the right moving sectors of closed string, the square length also is preserved i.e., $g_{\mu\nu} X_A^\mu X_A^\nu = G_{\mu\nu} \bar{X}_A^\mu \bar{X}_A^\nu$, where $A \in \{L, R\}$.

The equations (24) can be written as $\left(\begin{array}{c} p \\ \frac{1}{\alpha'} L \end{array} \right)^m = \Gamma^{mn} \left(\begin{array}{c} \Pi \\ \frac{1}{\alpha'} \Lambda \end{array} \right)_n$. Note that $\frac{1}{\alpha'} L^\mu$ and $\frac{1}{\alpha'} \Lambda^\mu$ are momenta of closed string in the dual spaces $\{\bar{X}^{\prime\mu}\}$ and $\{X^{\prime\mu}\}$, respectively. Therefore, the total momentum vectors in these 20-dimensional spaces also are related to each other by $O(10, 10; \mathbf{R})$ duality group.

The (10,10) Lorentzian momenta (p_L^μ, p_R^μ) , for $g_{\mu\nu} = \delta_{\mu\nu}$ and $\alpha' = \frac{1}{2}$, form an even self-dual Lorentzian lattice [11], i.e., the Lorentzian length is even

$$p_L^2 - p_R^2 = 2m^\mu n_\mu, \quad (31)$$

where $p_A^2 = G_{\mu\nu} p_A^\mu p_A^\nu$ and $A \in \{L, R\}$. To obtain the equation (31), we assumed that all directions of the spacetime to have the same radius of compactification. Note that all even self-dual (d, d) Lorentzian lattices are related by $O(d, d; \mathbf{R})$ rotations [13]. Since the momenta (p_L^μ, p_R^μ) transform as vectors under $O(10, 10; \mathbf{R})$ group, Hamiltonian in the closed string frame is invariant. More properties of the above duality group can be found in the Ref.[14].

Quantization

The quantization of the bosonic part gives the following equations

$$[\xi^\mu, \Pi^\nu] = [\xi'^\mu, \frac{1}{\alpha'} \Lambda^\nu] = i g^{\mu\nu}, \quad (32)$$

which can be written as

$$[x^\mu, p^\nu] = [x'^\mu, \frac{1}{\alpha'} L^\nu] = i G^{\mu\nu}. \quad (33)$$

Also the oscillators satisfy the relations

$$[\alpha_m^\mu, \alpha_n^\nu] = [\tilde{\alpha}_m^\mu, \tilde{\alpha}_n^\nu] = m \delta_{m+n,0} G^{\mu\nu}. \quad (34)$$

The quantization of the fermionic part leads to the relations

$$\begin{aligned} \{b_r^\mu, b_s^\nu\} &= \{\tilde{b}_r^\mu, \tilde{b}_s^\nu\} = \delta_{r+s,0} G^{\mu\nu}, \\ \{d_m^\mu, d_n^\nu\} &= \{\tilde{d}_m^\mu, \tilde{d}_n^\nu\} = \delta_{m+n,0} G^{\mu\nu}. \end{aligned} \quad (35)$$

where $(b_r^\mu, \tilde{b}_r^\mu)$ and $(d_n^\mu, \tilde{d}_n^\mu)$ are oscillators of the spinor $\bar{\psi}^\mu$ in the NS \otimes NS and R \otimes R sectors, respectively. The quantization of the worldsheet fields in the closed string frame, explicitly depends on the open string metric.

Note that for the variables in the coordinate system $\{X^\mu\}$, raising, lowering or contraction of indices can be done by the metrics $g_{\mu\nu}$ and $g^{\mu\nu}$, while for the variables in the closed string frame, such as $\{x^\mu, x'^\mu, p^\mu, L^\mu, \alpha_n^\mu, b_r^\mu, d_m^\mu, \dots\}$, indices can be raised, lowered or contracted by the open string metrics $G_{\mu\nu}$ and $G^{\mu\nu}$.

3 Boundary conditions of closed superstring

Now we develop the boundary state formalism for a D_p -brane with background field. Assume that the B -field has non-vanishing components only along the brane directions i.e., $B_{i\alpha} = B_{ij} = 0$ and $B_{\alpha\beta} \neq 0$. Therefore, the bosonic boundary state equations are

$$\begin{aligned} (G^{\alpha\beta}\partial_\tau X_\beta + \frac{1}{2\pi\alpha'}\theta^{\alpha\beta}\partial_\sigma X_\beta)_{\tau=0}|B_b\rangle &= 0, \\ (\delta X^i)_{\tau=0}|B_b\rangle &= 0. \end{aligned} \quad (36)$$

The set $\{X^\alpha\}$ shows the brane directions and the set $\{X^i\}$ shows the directions perpendicular to the brane. Since the ghost and the super-ghost parts of the boundary state are independent of the background field, we do not study them.

The boundary conditions on the fermionic degrees of freedom should be imposed on both R \otimes R and NS \otimes NS sectors. Worldsheet supersymmetry requires the two sectors to satisfy the boundary conditions

$$\begin{aligned} \left(G^{\alpha\beta}(\psi_{-\beta} - i\eta\psi_{+\beta}) - \frac{1}{2\pi\alpha'}\theta^{\alpha\beta}(\psi_{-\beta} + i\eta\psi_{+\beta})\right)_{\tau=0}|B_f, \eta\rangle &= 0, \\ (\psi_-^i + i\eta\psi_+^i)_{\tau=0}|B_f, \eta\rangle &= 0, \end{aligned} \quad (37)$$

where $\eta = \pm 1$ used to make GSO projection easily.

In terms of oscillators the bosonic part of the boundary state equations becomes

$$(G^{\alpha\beta}\Pi_\beta + \frac{1}{2\pi\alpha'^2}\theta^{\alpha\beta}\Lambda_\beta)|B_b\rangle = 0, \quad (38)$$

$$\Lambda^i|B_b\rangle = 0, \quad (39)$$

$$(\xi^i - y^i)|B_b\rangle = 0, \quad (40)$$

$$\left((G^{-1} - \frac{1}{2\pi\alpha'}\theta)^{\alpha\beta} a_{n\beta} + (G^{-1} + \frac{1}{2\pi\alpha'}\theta)^{\alpha\beta} \tilde{a}_{-n\beta} \right) |B_b\rangle = 0 , \quad (41)$$

$$(a_n^i - \tilde{a}_{-n}^i) |B_b\rangle = 0 . \quad (42)$$

The set $\{y^i\}$ indicates the transverse coordinates of the brane. The equations (36) and (37) reveal the effects of the noncommutativity of the brane on the closed superstring boundary state. The equation (38) describes the relation between the momentum (the momentum numbers) of the closed string and its winding numbers [10]. This equation implies that the noncommutativity and compactness of spacetime are coupled to each other. The lack of one of them leads to $\Pi^\alpha = 0$, which means that the emitted closed string propagates perpendicular to the brane. The second equation of (24) and the equation (38) give

$$p^\alpha = 0 , \quad (43)$$

that is, in the closed string frame, the closed string is emitted perpendicular to the brane. Also the first equation of (24) and the equation (39) lead to

$$L^i = 0 , \quad (44)$$

which means in the closed string frame, the closed string can not wrap around the compact direction \bar{X}^i .

The boundary state equations (36) and (37) in the closed string frame have the forms

$$\begin{aligned} (\partial_\tau \bar{X}^\alpha)_{\tau=0} |B_b\rangle &= 0 , \\ (\partial_\sigma \bar{X}^i)_{\tau=0} |B_b\rangle &= 0 , \\ (\bar{\psi}_-^\alpha - i\eta \bar{\psi}_+^\alpha)_{\tau=0} |B_f, \eta\rangle &= 0 , \\ (\bar{\psi}_-^i + i\eta \bar{\psi}_+^i)_{\tau=0} |B_f, \eta\rangle &= 0 . \end{aligned} \quad (45)$$

Apparently these equations have the form of the boundary state equations of the brane without background field. Since in mode expansions of these equations the oscillators depend on the background field, these equations describe the boundary conditions of a closed string emitted from a brane with background field. The equations (43) and (44) directly can be extracted from the first and the second equations of (45).

4 Conclusions

We found a compact coordinate system (i.e., $\{\bar{X}^\mu\}$) such that each coordinate and its T-dual coordinate are related to the spacetime coordinates and the T-dual coordinates of the

spacetime by the noncommutativity. Similar relations also hold for the fermions of the worldsheet $\{\bar{\psi}^\mu\}$. A novel feature is to cause the closed string state to have momentum which is a linear combination of its spacetime momentum and its winding numbers. In this frame the quantization of the worldsheet bosons and fermions is much similar to the zero B -field case that is, the closed string metric should be changed by the open string metric. This change also takes place for the supercurrent and the superconformal generators. We saw that the spacetime and the closed string frame are related to each other by $O(10, 10; \mathbf{R})$ duality rotation.

We observed the effects of noncommutativity and compactification in the boundary state equations of a closed superstring emitted from a noncommutative wrapped brane. In the closed string frame, the closed string propagates perpendicular to the brane. In this coordinate system the boundary conditions of the closed superstring appear similarly to the case without B -field.

References

- [1] N. Seiberg and E. Witten, JHEP **9909**(1999)032, hep-th/9908142.
- [2] A. Connes, M.R. Douglas and A. Schwarz, JHEP **9802**(1998)003, hep-th/9711162; M.R. Douglas and C. Hull, JHEP **9802**(1998)008, hep-th/9711165; F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, JHEP **9902**(1999)016, hep-th/9810072; A. Fayyazuddin and M. Zabzine, Phys. Rev. **D62**(2000)046004, hep-th/9911018; P.M. Ho and Y.T. Yeh, Phys. Rev. Lett. **85**(2000)5523, hep-th/0005159; Y.E. Cheung and M. Krog, Nucl. Phys. **B528**(1998)185, hep-th/9803031; D. Bigatti and L. Susskind, Phys. Rev. **D62**(2000)066004, hep-th/9908056; A. Schwarz, Nucl. Phys. **B534**(1998)720, hep-th/9805034; D. Kamani, Mod. Phys. Lett. **A17**(2002)237, hep-th/0107184; Europhys. Lett. **57**(2002)672, hep-th/0112153.
- [3] C.S. Chu and P.M. Ho, Nucl. Phys. **B550**(1999)151, hep-th/9812219; Nucl. Phys. **B568**(2000)447, hep-th/9906192; V. Schomerus, JHEP **9906**(1999)030, hep-th/9903205; F. Ardalan, H. Arfaei and M.M. Sheikh-Jabbari, Nucl. Phys. **B576**(2000)578, hep-th/9906161; N. Ishibashi, hep-th/9909176.
- [4] M.B.Green and P.Wai, Nucl. Phys. **B431** (1994) 131; C. Schmidhuber, Nucl. Phys. **B467** (1996) 146, hep-th/9601003; M. B. Green and M. Gutperle, Nucl. Phys. **B476** (1996) 484, hep-th/9604091; M. Billo, P. Di Vecchia and D. Cangemi, Phys. Lett.

- 400B**(1997) 63, hep-th/9701190; F. Hussain, R. Iengo and C. Nunez, Nucl. Phys. **B497**(1997)205, hep-th/9701143; O. Bergman, M. Gaberdiel and G. Lifschytz, Nucl. Phys. **B509** (1998)194, hep-th/9705130; M. Billo, P. Di Vecchia, M. Frau, A. Lerda, I. Pesando, R. Russo and S. Sciuto, Nucl. Phys. **B526**(1998)199, hep-th/9802088; G. Leigh, Mod. Phys. Lett. **A428**(1989)2767; C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys. **B288** (1987)525-550; S.A. Yost, Nucl. Phys. **B321**(1989)629-652.
- [5] S. Gukov, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **B423**(1998) 64, hep-th/9711112; T. Kitao, N. Ohta and J. Ge Zho, Phys. Lett. **B428**(1998)68, hep-th/9801135; C.G. Callan and I.R. Klebanov, Nucl. Phys. **B465**(1996)473, hep-th/9511173; M. Billo, P. Di Vecchia, M. Frau, A. Lerda, R. Russo and S. Sciuto, hep-th/9805091; H. Arfaei and D. Kamani, Nucl. Phys. **B561**(1999)57-76, hep-th/9911146; Phys. Lett. **B475**(2000)39, hep-th/9909079; D. Kamani, Phys. Lett. **B487**(2000)187, hep-th/0010019; Mod. Phys. Lett. **A15**(2000)1655, hep-th/9910043; Nucl. Phys. **B601** (2001)149-168, hep-th/0104089.
- [6] M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Phys. Lett. **400B**(1997)52, hep-th/9702037.
- [7] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Nucl. Phys. **B507**(1997)259, hep-th/9707068.
- [8] M. Li, Nucl.Phys. **B460**(1996)351-361 , hep-th/9510161.
- [9] C.G. Callan, C. Lovelace, C.R. Nappi and S.A. Yost, Nucl. Phys. **B308** (1988)221-284.
- [10] H. Arfaei and D. Kamani, Phys. Lett. **B452**(1999)54, hep-th/9909167.
- [11] K.S. Narain, Phys. Lett. **B169**(1986)369.
- [12] K.S. Narain, M.H. Sarmadi and E. Witten, Nucl. Phys. **B289**(1987)414.
- [13] J.P. Serre, A Course in Arithmetic (Springer-Verlag, New York, NY, 1973).
- [14] A. Giveon, M. Porrati and E. Rabinovici, Phys. Rep. **244**(1994)77-202.